The sixth order terms in (4) and (5) only enter in (10) into the terms of seventh and higher order.

3. Discussion

In the case of cylindrical electrodes ($p_i = q_i = 0$) the final results of **2** transform into the subsequent terms of the well known series expansion of the logarithmic potential

$$V(u, v) = V(0, 0) + u - \tfrac{1}{2} u^2 + \tfrac{1}{3} u^3 - \tfrac{1}{4} u^4 + - \dots .$$

Also in the case of the spherical condenser we get the initial terms of the corresponding potential.

The coefficients $a_{k,l}$ with $l \geqq 2$ contain no numerical constants and are either symmetrical (even terms in d) or antisymmetrical (odd terms in d) in p and q.

The potential of the electrodes is found from the v^0-terms in (1) by substituting $u = \pm d$:

$$V_{1,2}/E_0 \, r_e = a_{00} \pm a_{10} \, d + a_{20} \, d^2 \pm a_{30} \, d^3 + \dots .$$

The shape of electrodes, necessary to produce a desired type of field can be found by substituting (10) and putting $\varphi_e = \text{constant}$.

The parameters c and R_e' used in ion optical calculations [1] are connected with our parameters p and q through

$$c = r_e / R_e = -2 \, a_{02} / a_{10} ,$$

$$R_e' = (a_{02} \, a_{20} - \tfrac{1}{2} a_{10} \, a_{12}) / a_{02}{}^2 .$$

In exactly the same way the potential distribution can be found in rotational symmetric electrode systems, without a median plane of symmetry. In this case one has to take into account in (1) the odd terms in v. These terms will give results corresponding to the series expansions in [3], where the same series (1) is considered with $l = \text{odd}$.

The author is indebted to Prof. Dr. J. Kistemaker for his stimulating interest in this work, and to Mr. H. A. Tasman for valuable discussions.

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A modified Ion Slit Lens for virtual Variation of Slit Widths

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(Z. Naturforschg. **15 a**, 350—355 [1960]; eingegangen am 13. Februar 1960)

It is shown that a slit lens system, already known in literature and used in mass spectrometer collector systems to vary the resolving power, can be greatly improved by introducing a potential on one of the electrodes, which previously was at zero potential. The focusing at the collector is unaltered within optical aberrations of the third order and the range of adjustment is increased as compared with the original system. Experimentally it was found that the virtual collector slit width could be adjusted from 1 mm down to 0.15 mm, maintaining a fair peak shape.

In 1953 Craig [1] published a slit lens system for use in mass spectrometer collector systems. It consists of four slits, as shown in Fig. 1. The first, third and fourth electrodes are at zero potential. The potential of the second electrode is varied. The first three electrodes act as an ion lens, whose strength is controlled in this way. Indeed Craig succeeded in getting an adjustable resolving power, corresponding to virtual collector slit widths ranging from 1.00 mm down to 0.25 mm.

At the higher voltages on the second electrode however, the peak shape is badly impaired. This effect is caused by the change in the location of the image of the first slit, when through the variation of the potential at the second electrode the strength of the lens is varied. For a proper action of the slit system this image should coincide with the fourth electrode. In this case the fourth slit will act as an

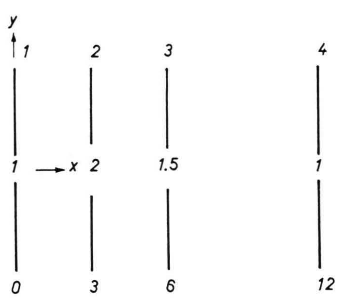

Fig. 1. The slit system.

[1] R. D. Craig, Use of a retarding slit in mass spectrometer collector systems, Reports on the conference on applied mass spectrometry, London 1953, p. 230; Metro Vick Research Report No. 4987.

entrance slit, if the image of the first slit is broader than this slit, thus producing a virtual decrease of the width of the first slit.

In the next sections it is proved, that by applying an adjustable potential to the third electrode, it is possible to vary the magnification of the lens, whereas the location of the image remains unchanged. The magnification can be varied widely, as well as the resolving power.

1. The equations for the ion trajectories

We suppose the first and fourth electrodes of the slit system to be at earth potential, which we denote by V_0, in agreement with the convention in ion optics to take the zero of potential equal to the potential at which the ions have zero kinetic energy. Let the potentials at the second and third electrodes be V_2 and V_3.

As has been shown by TIMM [2] the potential distribution along the axis of a slit lens system may be represented in fair approximation by a broken linear function. We devide the lens in five sections; in each section we assume a linear potential variation as indicated in Fig. 2. In accordance with the lens, pro-

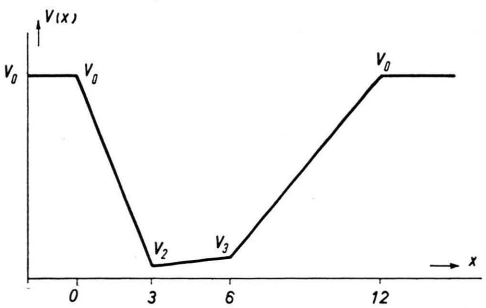

Fig. 2. The approximate potential along the axis of the slit system.

posed by CRAIG, the distances between the very thin electrodes are taken respectively as 3, 3, and 6 mm, so we get the approximate potential distribution:

$$
\begin{aligned}
-\infty &< x \leqq 0, & V &= V_0, \\
0 &\leqq x \leqq 3, & V &= \tfrac{1}{3}(V_2 - V_0)\, x + V_0, \qquad (1)\\
3 &\leqq x \leqq 6, & V &= \tfrac{1}{3}(V_3 - V_2)\, x + 2\,V_2 - V_3, \\
6 &\leqq x \leqq 12, & V &= \tfrac{1}{6}(V_0 - V_3)\, x + 2\,V_3 - V_0, \\
12 &\leqq x < \infty, & V &= V_0.
\end{aligned}
$$

[2] U. TIMM, Z. Naturforschg. **10 a**, 593 [1955].
[3] A. J. H. BOERBOOM, Thesis, Leiden 1957, p. 60.

In a region of constant potential the ion trajectories are straight lines, in the homogeneous fields between the electrodes the trajectories are parabolas. So in the different regions the ion trajectories are generally represented by

$$y = A_c\, x + B_c,$$

respectively
$$y = A_h \sqrt{V} + B_h, \qquad (2)$$

where all A and B are constant in each region.

Now the parameters A and B for each region have to be chosen in such a way, that the paths in the subsequent regions join and represent a possible ion trajectory.

Again TIMM [2] showed that this requires

$$y \quad \text{to be continuous}$$

and
$$2\,V\,\Delta y' + y\,\Delta V' = 0$$

at the breaks in the potential curve at $x = 0$, 3, 6, and 12. Here $\Delta y'$ and $\Delta V'$ denote the change in the derivatives y' and V' at the breaks.

In the same paper a matrix method is presented for the calculation of the imaging properties. This method will be used in the present paper. A conventional method giving the same results is used by the author in his thesis [3].

The straight ion trajectories before or beyond the lens are represented by the vector $\binom{y}{y'}$, where y has to be taken at the respective lens boundaries. The change of the vector of the trajectory at a break is given by the matrix

$$\begin{pmatrix} 1 & 0 \\ -\dfrac{\Delta V'}{2\,V} & 1 \end{pmatrix}.$$

The change of this vector over a region of homogeneous electric field is given by

$$\begin{pmatrix} 1 & \dfrac{2\,V_A}{V'}\left\{\sqrt{\dfrac{V_B}{V_A}} - 1\right\} \\ 0 & \sqrt{\dfrac{V_A}{V_B}} \end{pmatrix}$$

where V_A and V_B denote the potentials on the first and last electrodes relative to the moving ion at the field boundaries.

We denote

$$\sqrt{V_0} = u; \quad \sqrt{V_2} = w \quad \text{and} \quad \sqrt{V_3} = v.$$

So in our case the subsequent matrices are respectively

Break at $x=0$ $\quad M_1=\begin{pmatrix}1 & 0\\ \dfrac{u^2-w^2}{6\,u^2} & 1\end{pmatrix}$

$x=3$ $\quad M_3=\begin{pmatrix}1 & 0\\ \dfrac{2\,w^2-u^2-v^2}{6\,w^2} & 1\end{pmatrix}$

$x=6$ $\quad M_5=\begin{pmatrix}1 & 0\\ \dfrac{3\,v^2-2\,w^2-u^2}{12\,v^2} & 1\end{pmatrix}$

$x=12$ $\quad M_7=\begin{pmatrix}1 & 0\\ \dfrac{u^2-v^2}{12\,u^2} & 1\end{pmatrix}$

Segment $0<x<3$ $\quad M_2=\begin{pmatrix}1 & \dfrac{6\,u}{u+w}\\ 0 & \dfrac{u}{w}\end{pmatrix}$

$3<x<6$ $\quad M_4=\begin{pmatrix}1 & \dfrac{6\,w}{v+w}\\ 0 & \dfrac{w}{v}\end{pmatrix}$

$6<x<12$ $\quad M_6=\begin{pmatrix}1 & \dfrac{12\,v}{u+v}\\ 0 & \dfrac{v}{u}\end{pmatrix}$

These matrices provide the approximate shape of any ion trajectory through the lens.

2. The relation between V_2 and V_3

To avoid the multiplication of these seven matrices, we consider an ion trajectory in the middle region $3<x<6$, generally represented by

$$y=A\,\sqrt{V}+B\,. \tag{3}$$

This ion trajectory is represented by the vector $\begin{pmatrix}A\,w+B\\ \dfrac{v^2-w^2}{6\,w}A\end{pmatrix}$, when it enters the region, and by $\begin{pmatrix}A\,v+B\\ \dfrac{v^2-w^2}{6\,v}A\end{pmatrix}$, when it leaves the same region.

The trajectory of this ion in the object and image spaces is found by multiplication with resp. $M_1^{-1}\,M_2^{-1}\,M_3^{-1}$ and $M_7\,M_6\,M_5$.

For the following considerations it even would suffice to compute the elements of the first rows of these matrix products.

In this way one finds

$$Y_{\text{in}}=\begin{pmatrix}\dfrac{-u^2-v^2+u\,w+3\,w^2}{w\,(u+w)} & -\dfrac{6\,w}{u+w}\\ \dfrac{2\,u^3-2\,u^2\,w-4\,u\,w^2+2\,u\,v^2+3\,w^3-v^2\,w}{6\,u^2\,w} & \dfrac{2\,u\,w-w^2}{u^2}\end{pmatrix}\begin{pmatrix}A\,w+B\\ A\,\dfrac{v^2-w^2}{6\,w}\end{pmatrix},$$

$$Y_{\text{out}}=\begin{pmatrix}\dfrac{-u^2+u\,v+4\,v^2-2\,w^2}{v\,(u+v)} & \dfrac{12\,v}{u+v}\\ \dfrac{-2\,u^3+2\,u^2\,v-4\,u\,w^2-4\,v^3+6\,u\,v^2+2\,v\,w^2}{12\,u^2\,v} & \dfrac{2\,u\,v-v^2}{u^2}\end{pmatrix}\begin{pmatrix}A\,v+B\\ A\,\dfrac{v^2-w^2}{6\,v}\end{pmatrix}.$$

Now we consider a particular ion, which passes through the middle of the first slit: $x=0$, $y=0$. For this ion A and B have to be chosen in such a way that

$$\frac{-u^2-v^2+u\,w+3\,w^2}{w\,(u+w)}\,(A\,w+B)-\frac{v^2-w^2}{u+w}\,A=0\,. \tag{4}$$

When this ion also passes through the middle of the fourth slit, a focusing of the first slit on the fourth one is performed. This is the case, if

$$\frac{-u^2+u\,v+4\,v^2-2\,w^2}{v\,(u+v)}\,(A\,v+B)+\frac{2\,v^2-2\,w^2}{u+v}\,A=0\,. \tag{5}$$

A non-trivial solution of the equations (4) and (5) exists only if the determinant of the coefficients of A and B is zero.

So the condition that the first slit is imaged on the fourth slit is given by the equation:

$$\begin{aligned}u^4-u^3\,(v+w)-u^2\,(5\,v^2+2\,v\,w+2\,w^2)\\ -u\,(v^3-7\,v^2\,w-6\,v\,w^2+2\,w^3)-6\,v^4\\ +2\,v^3\,w+24\,v^2\,w^2+4\,v\,w^3-8\,w^4=0\,.\end{aligned} \tag{6}$$

This is a homogeneous equation in u, v, and w. Dividing by u^4 we get a condition between v/u and w/u. This means that only these ratios are significant for the lens, in corraboration with the well known fact in ion optics, that only ratios of potentials are significant. Therefore we take $u=1$, and get for every value of w an equation of the fourth degree for v.

Now v^2 and w^2 are the potentials of the second and third electrodes. When $0\leqq v^2\leqq 1$ and $0\leqq w^2\leqq 1$, we can take these voltages from the

w	first root	second root	third root	fourth root
0.00	0.33333	−0.50000	$+i$	$-i$
0.10	0.32184	−0.52743	$0.03613+0.92733\,i$	$0.03613-0.92733\,i$
0.20	0.31395	−0.56187	$0.07396+0.80474\,i$	$0.07396-0.80474\,i$
0.30	0.29512	−0.60313	$0.12067+0.60092\,i$	$0.12067-0.60092\,i$
0.32	0.28404	—	—	—
0.34	0.26409	—	—	—
0.36	0.22042	—	—	—
0.38	0.0951406	—	—	—
0.39	0.00333597	—	—	—
0.39039	0.00000	—	—	—
0.40	−0.07486	−0.65099	$0.34625+0.24637\,i$	$0.34625-0.24637\,i$
0.45	−0.32953	−0.67765	$0.49526+0.16543\,i$	$0.49526-0.16543\,i$
0.48	−0.43716	−0.69486	$0.56267+0.07267\,i$	$0.56267-0.07267\,i$
0.50	−0.50000	−0.70711	0.50000	0.70711
0.60	−0.72429	−0.80878	0.46000	1.10640
0.70	−0.80447	−1.01471	0.47597	1.40988
0.80	−0.86914	−1.21822	0.50288	1.68456
0.90	−0.93395	−1.41175	0.53489	1.94415
1.00	−1.00000	−1.59806	0.57032	2.19440
2.00	−1.70680	−3.30867	1.00000	4.51547

Table 1.

accelerating voltage of the ion. Particularly in connection with the desired proportionality of all potentials in a mass spectrometer, this is convenient. So we permit for w the domain $0 \leqq w \leqq 1$ and solve the equation in respect to v. We get the figures of Table 1, and we see that most values of w between 0 and 1 provide a root v between 0 and 1.

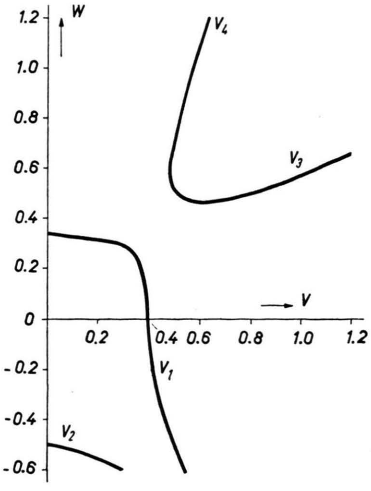

Fig. 3. The four roots of equation (6).

Our computations were only a rough approximation. An exact calculation would modify these values *. The general trend, however, will persist and thus it is possible to image the first slit on the fourth one.

Through some general considerations we can show that only aberrations of the third order occur in this imaging: Any ion trajectory is defined by the quantities A and B in (3). The central line of the slit system is a line of symmetry. When we replace A and B through $-A$ and $-B$, the second ion trajectory will be the mirror of the first one. This implies, that in the series expansion of the ion trajectory to A and B only terms of odd degree in A and B together can enter, so A, B, A^3, A^2B, AB^2, and B^3. First order focusing occurs, when the terms of the first degree vanish. Then in general the four terms of the third degree remain, representing the third order aberrations.

3. The magnification

As all points with $x=0$ are imaged on points with $x=12$, the ratio of the values of $y(12)$ and $y(0)$ of any ion trajectory furnishes the magnification. The trajectory with $A=1$ and $B=0$ provides the simple expression

$$M=\frac{u+w}{u+v}\,\frac{u^2-u\,v-6v^2+4\,w^2}{u^2-u\,w+2\,v^2-4\,w^2}$$

and the selected combinations of v and w give the values of Table 2. These figures are plotted in Fig. 4.

* The exact potential distribution in the ion lens of Fig. 1 is computed by the author, Z. Naturforschg. **14 a**, 809 [1959].

w	v	M	w	v	M
0.00	0.33333	0	0.60	0.46000	−1.262
0.10	0.32184	0.075	0.70	0.47597	−1.073
0.20	0.31395	0.278	0.80	0.50288	−0.995
0.30	0.29512	1.059	0.90	0.53489	−0.959
0.32	0.28404	1.527	1.00	0.57032	−0.942
0.34	0.26409	2.452			
0.36	0.22042	5.127	0.50	0.70711	−3.000
0.38	0.09514	29.7	0.60	1.10640	−3.242
0.39	0.00334	1371	0.70	1.40988	−3.161
0.39039	0.00000	∞	0.80	1.68456	−3.064
0.40	—	—	0.90	1.94415	−2.976
0.50	0.50000	−2.000	1.00	2.19440	−2.901

Table 2.

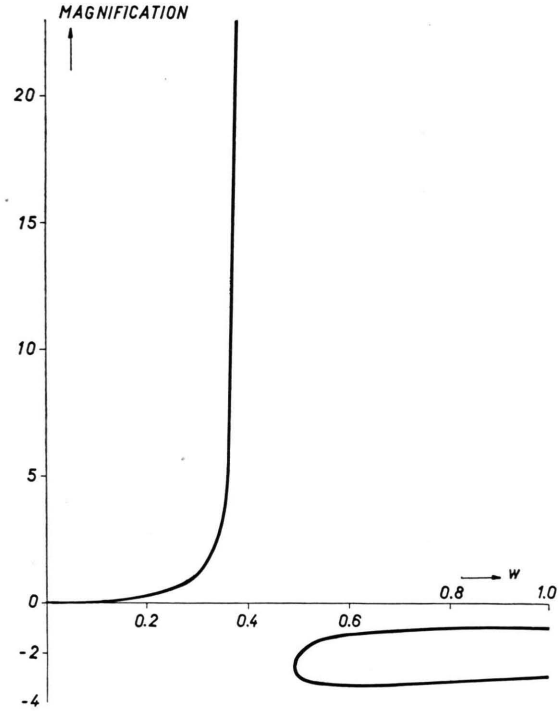

Fig. 4. The magnification.

Denoting by

- s_1 the source slit width,
- s_2 the collector slit width (first electrode of the slit system),
- s_3 the width of the fourth electrode,
- R the radius of curvature of the ion trajectories in the magnetic field,

the conventional formula for the resolving power

$$\text{res. pow.} = 2\,R/(s_1 + s_2)$$

is modified into

$$\text{res. pow.} = 2\,R/(s_1 + s_3/M)$$

as long as $$s_3/M \leqq s_2\,.$$

However, s_3 and M should be chosen in such a way, that $s_3/M \geqq s_1$ in order to maintain the maximum peak height.

4. Experimental

A slit lens was constructed of the design of Fig. 1, with electrodes of $\frac{1}{2}$ mm thickness. As this lens was used on a 180° mass spectrometer, the second electrode was cut to allow the application of an electric cross field to compensate the action of the magnetic stray field. To some extent the experiments will be influenced, but the general trend of the results should remain.

The accelerating voltage of the ions was 2000 V and the whole range of voltages V_2 and V_3 between 0 and

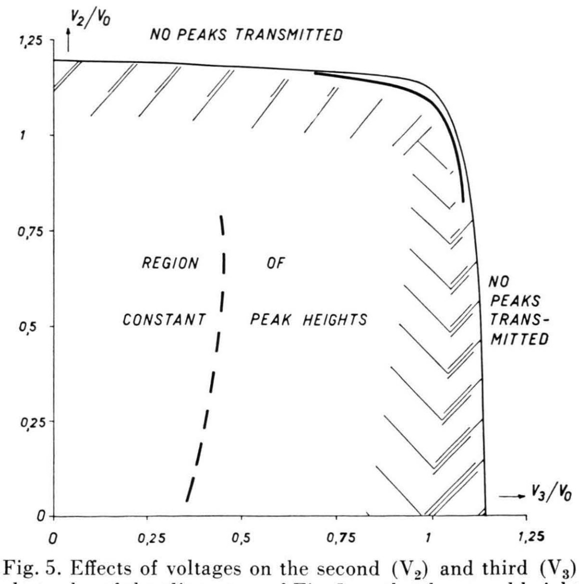

Fig. 5. Effects of voltages on the second (V_2) and third (V_3) electrodes of the slit system of Fig. 1 on the shape and height of the peaks.

— Imaging condition
\ Regions of decreasing peak heights
/ Regions of varying peak heights
// Double peaks
/// Threefold peaks.

2500 V on the second and third electrodes was researched. Here the earth potential is again taken to be zero.

RbCl was used in a thermal ionisation source to provide ions of mass 85 and 87. The spectra were checked on loss in peak height, peak shape and resolving power. To eliminate the influence of retardation in the recorder, optical aberrations in the mass spectrometer and pressure broadening of the peaks, the base and top widths of the peaks were measured at 5% and 95% of the peak height and the peak shape idealized to the theoretical trapezoid.

From it, the virtual collector and source slit widths were calculated. If the first slit is imaged on the fourth slit, a back imaging of the fourth slit on the first one will provide the actual width of the source slit. Any defocusing will increase this value.

Indeed at a constant value of V_3, the virtual source slit width shows a minimum equal to the real source slit width at a certain value of V_2. At the lower values of V_3 this minimum is very broad and almost independent of V_2. This corresponds with the root v_3. At the higher voltages, however, there is a critical relation between V_2 and V_3, the peaks dropping sharply to zero at the one side, and showing a bad shape and loss in height at the other side. This corresponds with the root v_1. A general review of the experimental results is presented in Fig. 5.

Along the latter part of the imaging curve the resolving power can be increased up to 600 and virtual collector slit widths down to 0.15 mm can be realised.

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Zur Theorie der Spinoren im Riemannschen Raum

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Die Theorie der Spinoren im RIEMANNschen Raum von INFELD und VAN DER WAERDEN wird mit Hilfe der bereits in der projektiven Relativitätstheorie bewährten Methode der Basisvektoren weiterentwickelt. Die spinorielle Geometrie läßt sich in eleganter Weise bis zur Ableitung von grundlegenden Identitäten analog den BIANCHI-Identitäten der tensoriellen Geometrie aufbauen. Es ergeben sich mehrere wichtige Beziehungen für den gekrümmten Spinorraum. Während in der üblichen Theorie die Wurzel des elektromagnetischen Feldes im Nichtverschwinden der kovarianten Ableitung des metrischen Spinors gesehen wird, da sich keine andere Interpretationsmöglichkeit zu bieten scheint, kann gezeigt werden, daß auch die wesentlich vereinfachte Geometrie mit verschwindender kovarianter Ableitung des metrischen Spinors zwangsläufig auf einen antisymmetrischen Tensor führt, der dem elektromagnetischen Feld zugeordnet wird.

Seit erkannt wurde, daß zum Aufbau einer Theorie der Elementarteilchen den Spinoren als den gegenüber den Tensoren elementareren Gebilden der Vorzug gegeben werden muß, und außerdem das Problem der Formulierung derartiger Theorien in beliebigen Koordinaten aufgetreten ist, um einer eventuellen Erfassung der Gravitonen Raum zu geben, ist das Interesse für eine elegante Theorie der Spinoren im RIEMANNschen Raum gestiegen. Bekanntlich wurde die DIRAC-Gleichung auf zwei verschiedenen Wegen für die RIEMANNsche Geometrie verallgemeinert, nämlich von FOCK und IWANENKO [1] und von INFELD und VAN DER WAERDEN [2]. Aus verschiedenen Gründen gibt Verfasser dem Apparat von INFELD und VAN DER WAERDEN den Vorzug, obwohl der Matrizen-Formalismus relativ häufiger in der Literatur [3] verwendet wird, da jener Apparat den Transformationsmechanismus im Tensor- und Spinorraum sehr klar hervortreten läßt. In mehreren Arbeiten [4] konnte Verfasser zeigen, daß sich die Methode der tensoriellen Basisvektoren zur Formulierung der projektiven Relativitätstheorie gut eignet. Deshalb wird diese Methode auf spinorielle Basisvektoren verallgemeinert, die zur Untersuchung der Geometrie des gekrümmten Spinorraumes verwendet werden sollen.

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